Constrained Models on Ammann Beenker Tilings

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Some of the most important phenomena in physics arise when strong correlations emerge from local constraints. Examples include dimer models (tiling a chess board with dominoes) and emergent 'magnetic monopole' excitations in crystals called spin ices. We outline results for a range of constrained models in a new setting: aperiodic Ammann Beenker tilings (AB). These lesser-known cousins of Penrose tilings have the symmetries of certain layered quasicrystals.

We prove the existence of Hamiltonian cycles (visiting each vertex precisely once), and thereby solve a range of related problems including the three-colouring problem and the travelling salesperson problem [1]. Potential applications include adsorption, catalysis, and scanning tunneling microscopy. Using machine learning (RSMI-NE) we identify an emergent discrete scale invariance to the structure of dimer matchings [2,3]. In ongoing work, we apply DMRG to the quantum dimer model in this (2D) system.

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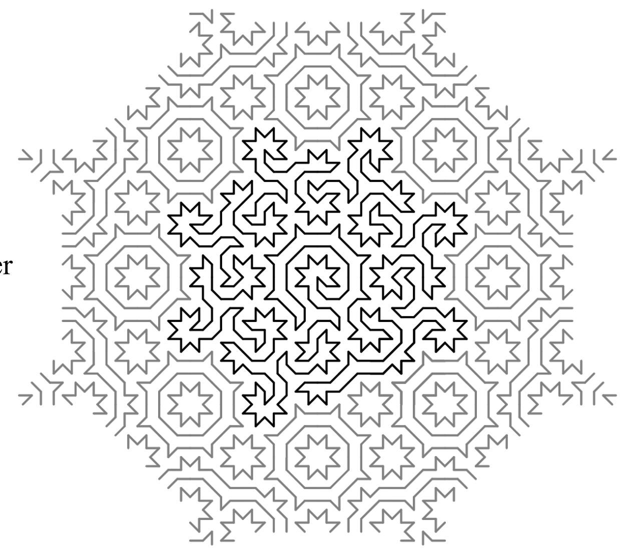

Fig.1: A Hamiltonian cycle on a subset of sites of an Ammann Beenker tiling describes the most efficient route an STM can take.

The hat tiling is topologically conjugate to a model set

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In a recent preprint [1], it was shown that a certain non-convex polygon (the hat), along with its mirror image, can tile the plane, but only non-periodically. This tile was therefore called a monotile, or an einstein. In any hat tiling, all tiles have coordinates in a triangular lattice. Hats can be combined to certain clusters, called meta-tiles, and there is a combinatorial inflation symmetry present in these meta-tile tilings, leading to a hierarchical structure and aperiodicity. Moreover, it is shown that the meta-tiles can be deformed in such a way that the combinatorial inflation becomes a true geometric inflation, with a scaling factor τ^2.

In this talk, we show that this shape change of the meta-tiles is asymptotically negligible in the sense of [2], meaning that it does not mess up the long-range aperiodic order of the system. Hence, the original meta-tile tiling and the deformed meta-tile tiling with the true τ^2 inflation form topologically conjugate dynamical systems under the translation action. As a result, their dynamical spectra coincide, which implies, in particular, that their reciprocal lattices are the same. Moreover, by the overlap algorithm [3] we can show that both tilings have pure-point dynamical and diffraction spectra.

Inflation systems with pure-point spectrum are expected to be cut-and-project sets, also known as model sets. This is the case also for the self-similar meta-tile tiling, for which we can construct a cut-and-project scheme based on a lattice with triangular symmetry and τ^2 scaling. With a proper choice of control points of the meta-tiles, the set of control points of the tiling can be described as a two-component model set with two fairly simple windows, regular triangles with a triangular hole. Only the interior boundaries between different tile types are fractal.

In summary, we show that the hat tiling is topologically conjugate to the self-similar meta-tile tiling, which in turn is a genuine cut-and-project tiling with reasonably simple windows and pure-point spectrum.

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