Fascinating aperiodic crystals*

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For more than a century the understanding of physical and chemical properties of solids has been based on the notion of periodicity in a crystal, associated with long-range order. The discovery, in a wide range of systems, of solids with long-range order yet without translational periodicity, at least in one dimension, has led to a new definition of crystals. This new class of ordered structures, called aperiodic crystals, encompasses incommensurately modulated phases, incommensurate composites and quasicrystals. It has opened a completely new field of research where the understanding of the atomic structure and associated physical and chemical properties has to be reconsidered with new perspectives.

This lecture will provide an introduction to the fascinating field of aperiodic crystals focussing on icosahedral quasicrystals. I will present results going from the detailed understanding of the atomic structure to some of their physical properties. This field is a nice example where the interplay between the development of experimental techniques and advances in the theory has been crucial.

* This introductory lecture, which will be given at the Steinhardt Museum of Natural History at 14:00, is delivered jointly with the *Tel Aviv University Physics Colloquium*, and is open to members of the Tel Aviv University community. Members of the community are invited to stick around for two tutorials, which will follow the introductory lecture – one on almost periodicity, and another on topological quantum numbers in quasicrystals.

Tutorial 1: Almost periodicity

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In this basic tutorial, I will introduce a variety of different notions of almost-periodic functions and measures. In particular, I will explain the difference between periodic, limit-periodic, quasiperiodic, limit-quasiperiodic, Bohr, mean, Besicovitsch and Weyl almost-periodic functions. This can be done via their Fourier-Bohr coefficients or different choices of the (semi)norm of a function, which lead to different sets of almost periods. The tutorial will include many examples, and will provide much of the background, necessary for following modern literature on the mathematics of aperiodic long-range order [1] and the nature of pure-point or Bragg-peak diffraction [2].

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[1] M. Baake, U. Grimm, *Aperiodic Order: Volume 1, A Mathematical Invitation* (Cambridge University Press, 2013).
[2] D. Lenz, T. Spindeler, N. Strungaru, arXiv:2006.10821.

Tutorial 2: Topological quantum numbers in quasicrystals

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In this tutorial lecture we explain the mathematical background for the construction of topological quantum numbers in aperiodic crystals. Often topological quantum numbers are referred to as Chern numbers, as they arose first as Chern numbers of vector bundles defined by solutions of the Schrödinger equation for periodic crystals. We explain how to define them in a non-commutive way which is also applicable to aperiodic crystals. We discuss the relevance of topological quantum numbers in quasicrystals. In this context we explain a particular manifestation of the bulk boundary correspondence which relates the labeling of gaps in the spectrum of a one-dimensional quasiperiodic Hamiltonian to the phason motion [1].

[1] J. Kellendonk, E. Prodan. *Annales Henri Poincaré*, **20** (2019) 2039-2070.