Pure point diffraction and almost periodicity

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We first review the framework of mathematical diffraction theory. We then discuss notions of almost periodicity and turn to recent results obtained in joint work with Timo Spindeler and Nicolae Strungaru [1,2]. These results characterize pure point diffraction via almost periodicity features of the underlying structure. In particular our results characterize structures that have pure point diffraction together with well-defined phases.

[1] Daniel Lenz, Timo Spindeler, Nicolae Strungaru, "*Pure Point Diffraction and Mean, Besicovitch and Weyl Almost Periodicity*", Preprint (2020) arXiv:2006.10821.
[2] Daniel Lenz, Timo Spindeler, Nicolae Strungaru, "*Pure point spectrum for dynamical systems and mean almost periodicity*", Preprint (2020) arXiv:2006.10825.

Local order constraints on pure-point diffraction

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We consider the diffraction on families of atomic structures obeying common local rules in a wide sense of the term (in particular, including disordered systems such as models of decorated random tilings). It is shown that imposing such rules results in constraints on the pure point part of the diffraction measure, taking the form of linear equations on the partial amplitudes. These equations depend on the properties of the corresponding prototile space of the local rules – a geometric object encoding the local order of the structure. Whenever Bragg peaks fill densely a linear subspace of the reciprocal space, for almost all of them, with the possible exception of a nowhere dense subset, the coefficients of these equations *depend smoothly on the physical wave vector*. For a given prototile space, these coefficients can be calculated explicitly in terms of finite trigonometric sums.

[1] P. Kalugin, and A. Katz. "Constraints on pure point diffraction on aperiodic point patterns of finite local complexity." Journal of Physics A: Mathematical and Theoretical **55** (2022): 065203.

Analysis of hyperuniformity reveals small statistical approximant patches in quasicrystals

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Recently two of us have introduced a new method, based on the Generalized Dual Method (GDM), for the efficient creation of quasicrystalline patterns with arbitrary N-fold rotational symmetry [1]. The method can create perfect quasiperiodic environments at any given point in the space. Therefore, we are able to analyze the statistics of large parts of quasicrystalline patterns that are not necessarily close to the center of symmetry of the system which is essential for a good sampling as for large N the environment around the symmetry center can be very different from any environment further away from it.

We use the new method to study hyperuniformity in quasicrystals with large rotational symmetry. Hyperuniform systems are characterized by anomalously suppressed long-wavelength (i.e., large-length-scale) density fluctuations compared to those found in ordinary gases and fluids as well as in amorphous solids. We show that all quasicrystals that we have analyzed are hyperuniform. Furthermore, the degree to which the large-scale fluctuations are suppressed reveals a characteristic length-scale that approximately growth like N^2. We argue that the two-particle statistics of patches of the quasicrystal with the size of this length resemble the statistics of the complete quasicrystal. Thus, we term these patches *statistical approximant patches*. Note that typically considered approximants roughly grow exponentially with the rank and thus N. As a consequence, for large N statistical approximant patches are much smaller than classical approximants and might facilitate the sampling that is needed to determine statistical properties like the pair correlation function.

[1] A.R.M. Sosa, A.S. Kraemer, Journal of Physics A: Mathematical and Theoretical **55** (2022), 245001.

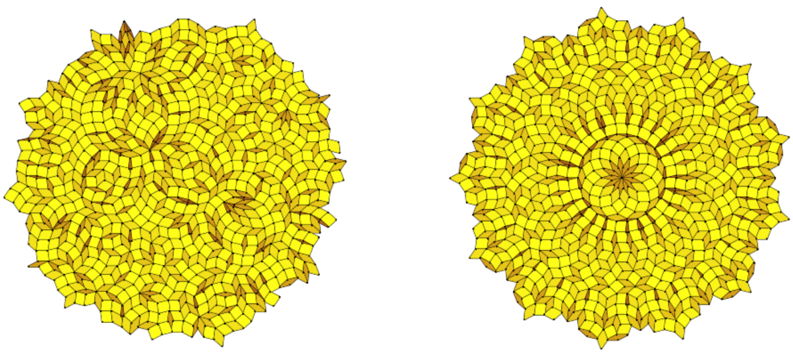

Fig.: Patches of a quasicrystalline pattern with 13-fold rotational symmetry, close to the symmetry center (right hand side) and far away from the symmetry center (left hand side).

The Lorentz gas on quasicrystals

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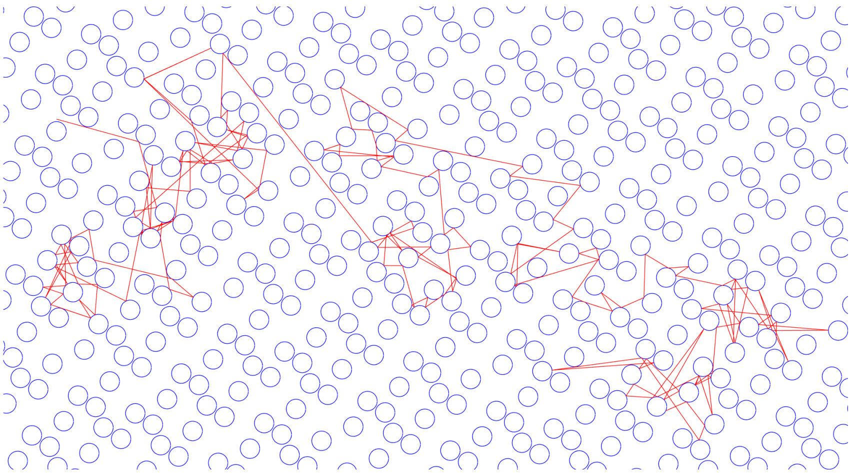
The Lorentz gas was introduced at the beginning of the 20th century to describe electron motion in metals. This system was thoroughly studied by many people over the 20th century. In particular, starting with Sinai's work in 1969, the *periodic* Lorentz gas in two dimensions led the way to the development of many tools in modern ergodic theory, and by now the periodic Lorentz is very well understood in two dimensions [1]. In contrast, all properties of the *aperiodic* Lorentz gas were essentially unknown until the work of Marklof-Strömbergsson [2] from about a decade ago, who studied the so-called Boltzmann-Grad limit in the quasicrystal case.

In this talk I will focus on recent rigorous results ([3] and [4]), joint with Agnieszka Zelerowicz, where we study the aperiodic Lorentz where local scatterer configurations are defined by the local patterns of a repetitive aperiodic tiling of finite local complexity. This large class includes quasicrystals, and we focus on the behavior of this system in the finite horizon setting, that is, in the case far from the Boltzmann-Grad limit.



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[1] N. Chernov, R. Markarian, AMS Mathematical Surveys and Monographs **127** (2006)
[2] J. Marklof, A. Strömbergsson, Comm. Math. Phys. **330** (2014) 723-755.
[3] R. Treviño, A. Zelerowicz, Comm. Math. Phys. **396** (2022) 1305-1338.
[4] R. Treviño, A. Zelerowicz, *Aperiodic and linearly repetitive Lorentz gases of finite horizon are not exponentially mixing* (arxiv preprint 2203.07215, 2022, to appear in DCDS).